Kuwait University	Math 101	Date: May 3, 2007
Dept. of Math. & CS	2nd Exam	Duration: 90 min.

Calculators, mobile phones, pagers and all other mobile communication equipment are not allowed.

Answer the following questions:

1. Use differentials to approximate $\frac{1}{(0.98)^{10}}$. [3 pts.]

2. a) State the Mean Value Theorem [1 pt.]

   b) Let

$$f(x) = \frac{x+1}{x-1}.$$

   Show that there is no real number c such that

$$f(2) - f(0) = 2f'(c).$$

   Why does this not contradict the Mean Value Theorem? [3 pts.]

3. A metal in the shape of a right circular cone, whose height is twice its radius, is being heated. If the radius is increasing at a rate of 0.001 cm /sec., find the rate at which its volume is increasing when its radius is 2 cm. [4 pts.]

4. Use implicit differentiation to find y'', if $x^2 + y^4 = 16$. [3 pts.]

5. Let $f(x) = 3x(x - \frac{5}{3})^{\frac{2}{3}}$ be defined on the interval $[-1, 2]$. Find the absolute maximum and absolute minimum of f. [3 pts.]

6. Let $f(x) = \frac{x^2 - 9}{2x - 4}$. [8 pts.]

   a. Find the x and y –intercepts of f.

   b. Find the vertical and horizontal asymptotes to the graph of f, if any.

   c. Find the intervals on which f is increasing and the intervals on which f is decreasing, if any.

   d. Find the intervals on which f is concave up and the intervals on which f is concave down, if any.

   e. Sketch the graph of the function f.

Total [25 pts.]

Total= 25pts

Answers

Q.1 $f(x) = \frac{1}{x^{10}}$; $f(1) = 1$, $f'(1) = -10$

$\frac{1}{(0.98)^{10}} \approx 1 - 10(0.98 - 1) = 1 - 10(-0.02) = 1.2$

Q2. (a) Statement of the MVT.

(b) $f'(c) = \frac{(c-1)(c+1)}{(c-1)^2} = \frac{-2}{(c-1)^2}$

$f(2) - f(0) = 3 - (-1) = 4 = 2f'(c)$

$\Rightarrow 4 = \frac{-4}{(c-1)^2} \Rightarrow (c-1)^2 = -1$ (which is not possible)

f is not differentiable on $(0,2)$.

Q3. $V = \frac{1}{3}\pi r^2 h$ where $h = 2r$.

$V = \frac{1}{3}\pi(2r^3)$

$\frac{dV}{dt} = 2\pi r^2 \frac{dr}{dt} = 8\pi(0.001)$ cm^3/sec.

Q4. $2x + 4y^3 y' = 0$

$y' = -2x/4y^3 = -\frac{x}{2y^3}$

$y'' = -\left[\frac{2y^3 \cdot 1 - x(6y^2)y'}{4y^6}\right]$

$= -\frac{\left[2y^3 - 6xy^2 \cdot \left(-\frac{x}{2y^3}\right)\right]}{4y^6} = -\left(\frac{2y^4 + 3x^2}{4y^7}\right)$.

Q5. $f'(x) = \frac{5x-5}{(x-5/3)^{1/3}}$, critical Nos. $\{1, 5/3\}$

c	$f(c)$	
-1	$-3(\frac{64}{9})^{1/3}$	— min
1	$3(\frac{4}{9})^{1/3}$	
$5/3$	0	
2	$3(\frac{8}{9})^{1/3}$	— max.

Q6. (a) y-intercept $(0, 9/4)$

x-intercept $(\pm 3, 0)$

(b) $x = 2$ V.A

No horizontal asy.

(c) $f'(x) = \frac{2x^2 - 8x + 18}{(2x-4)^2}$

(d) $f''(x) = -5/(x-2)^3$